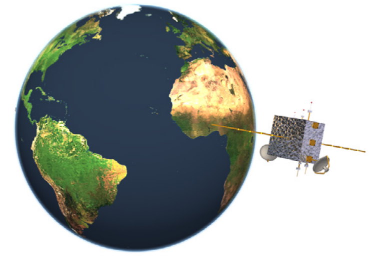
AP1 Gravity

1. A satellite of mass m_S orbits a planet of mass m_P at an altitude equal to twice the radius (R) of the planet. What is the satellite's speed assuming a perfectly circular orbit?



(A) $v=\sqrt{\frac{Gm_P}{R}}$

(B) $v=\sqrt{\frac{Gm_S}{R}}$

(C) $v=\sqrt{\frac{Gm_S}{2R}}$

(D) $v=\sqrt{\frac{Gm_P}{3R}}$

Answer: (D) $v=\sqrt{\frac{Gm_P}{3R}}$

Students must first recognize that the radius of the satellite's orbit is 3R, the radius of the planet plus the altitude of the satellite above the surface of the planet. Then, a force analysis recognizing the gravitational force of attraction provides a centripetal force yields:

$$F_{net_C}=F_g=ma_c=\frac{mv^2}{r}\rightarrow\frac{m_Sv^2}{3R}=\frac{Gm_Sm_P}{(3R)^2}\rightarrow v^2=\frac{Gm_P}{3R}\rightarrow v=\sqrt{\frac{Gm_P}{3R}}$$

EK: 3.C.1 Gravitational force describes the interaction of one object that has mass with another object that has mass.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 3.C.1.2 The student is able to use Newton's law of gravitation to calculate the gravitational force between two objects and use that force in contexts involving circular orbital motion.

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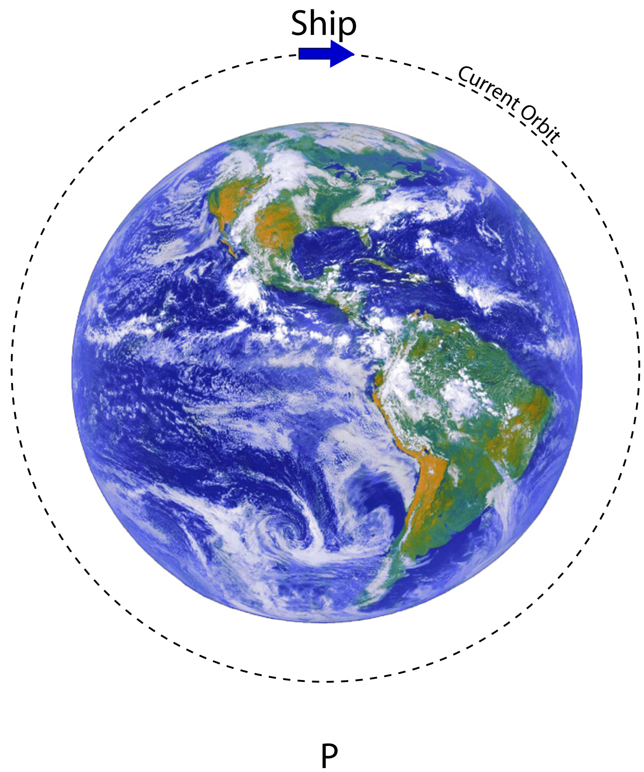
2. A spaceship in a circular orbit 400 km above the surface of the Earth wishes to manipulate its orbit to reach a point P on the opposite side of the Earth which is 1000 km above the Earth's surface. If the spaceship is at the position shown in the diagram and currently moving in a clockwise direction, in which direction should the ship accelerate in order to reach point P?

(A) toward the top of the page
(B) toward the right of the page
(C) toward the bottom of the page
(D) toward the left of the page



Answer: (B) toward the right of the page

To increase the radius of its orbit, the ship must attain a higher velocity, which requires an acceleration in the direction of its current velocity, or to the right of the page as depicted in this diagram. This will shift the orbit from a circular orbit to an elliptical orbit, and allow the ship to reach point P. (Note that a second acceleration will be required upon reaching point P if the ship wishes to maintain a circular orbit 1000 km above the Earth's surface.)

EK: 2.B.1 A gravitational field g at the location of an object with mass m causes a gravitational force of magnitude mg to be exerted on the object in the direction of the field. 3.B.1 If an object of interest interacts with several other objects, the net force is the vector sum of the individual forces.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 2.B.1.1 The student is able to apply F=mg to calculate the gravitational force on an object with mass m in a gravitational field of strength g in the context of the effects of a net force on objects and systems. 3.B.1.1 The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations with acceleration in one dimension.

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3. A rock is thrown horizontally from the top of a 100-meter-high vertical cliff on Planet Unicorn with a speed of 20 m/s. If the mass of Planet Unicorn is 10^{25} kg and the top of the cliff is approximately 4000 kilometers from the center of the planet, how far from the base of the cliff does the rock land?

(A) 0.022 m
(B) 0.044 m
(C) 43.8 m
(D) 90.1 m

Answer: (C) 43.8 m

The acceleration due to gravity is the gravitational field strength, which can be determined from Newton's Law of Universal Gravitation. The horizontal distance traveled by the rock is the time it takes for the rock to strike the ground (a kinematics exercise) multiplied by the horizontal velocity of the rock (given in the problem).

$$\Delta x = v_x t = (20\,{}^{m}\!/\!{}_{s})\sqrt{\frac{2h}{g}} = (20\,{}^{m}\!/\!{}_{s})\sqrt{\frac{2hr^2}{Gm}} = (20\,{}^{m}\!/\!{}_{s})\sqrt{\frac{2(100m)(4000000m)^2}{6.67\times10^{-11}\,{}^{N\bullet m^2}\!/\!{}_{kg^2}(10^{25}kg)}} = 43.8m$$

EK: 1.C.2 Gravitational mass is the property of an object or a system that determines the strength of the gravitational interaction with other objects, systems, or gravitational fields. 2.B.1 A gravitational field g at the location of an object with mass m causes a gravitational force of magnitude mg to be exerted on the object in the direction of the field. 4.A.2 The acceleration is equal to the rate of change of velocity with time, and velocity is equal to the rate of change of position with time.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 2.B.1.1 The student is able to apply F=mg to calculate the gravitational force on an object with mass m in a gravitational field of strength g in the context of the effects of a net force on objects and systems. 4.A.2.3 The student is able to create mathematical models and analyze graphical relationships for acceleration, velocity, and position of the center of mass of a system and use them to calculate properties of the motion of the center of mass of a system.

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4. Which of the following changes would increase the magnitude of the gravitational field intensity an object feels when near a planet? (Select two answers.)

(A) increase the mass of the object
(B) increase the mass of the planet
(C) decrease the spin rate of the planet
(D) decrease the separation distance between object and planet

Answers: (B) and (D)
By definition field intensity, $g = F_g/m_o$ where m_o is the mass of the object. This equation expands to $g = Gm_p/r^2$ leading to choices (B) and (D)

EK: 2.B.1 A gravitational field g at the location of an object with mass m causes a gravitational force of magnitude mg to be exerted on the object in the direction of the field. 2.B.2 The gravitational field caused by a spherically symmetric object with mass is radial and, outside the object, varies as the inverse square of the radial distance from the center of that object.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 2.B.1.1 The student is able to apply F=mg to calculate the gravitational force on an object with mass m in a gravitational field of strength g in the context of the effects of a net force on objects and systems. 2.B.2.1 The student is able to apply g=GM/r^2 to calculate the gravitational field due to an object with mass M, where the field is a vector directed toward the center of the object of mass M.

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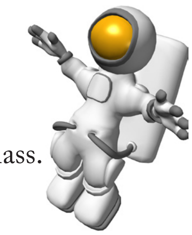
5. Marty is an astronaut who is preparing to go on a mission in orbit around the Earth. For health reasons, his mass needs to be determined before take-off and while he is in orbit. The morning of the launch, Marty sits on one pan of a two-pan scale and 94 kg of mass is needed to balance him.



(a) State and explain whether the two-pan scale registered Marty's gravitational mass or inertial mass.

(b) After a few days in orbit Marty is again to determine his mass. Explain why the two-pan scale used before launch cannot be used to measure his mass while in orbit.

(c) To determine Marty's mass in orbit he is to sit in a chair of negligible mass that is attached to a wall by a spring that has a force constant, k. Consequently, the chair freely vibrates back and forth with a period, T when displaced sideways a distance, x. Explain how the spring-mounted chair can be used to determine Marty's mass, m. Give relevant measurements and equation(s).

(d) If Marty has lost mass while in orbit, what specific change would occur when he sits in the chair and starts it oscillating?

(e) Explain why this spring-mounted chair measures Marty's inertial mass.

Answer:

(a) Gravitational mass. The pans of the scale balance under the influence of gravity, not any other force.
(b) In orbit the effects of gravity are not felt because everything is in free-fall. Consequently the scales will not become balanced or unbalanced when objects are placed on them.
(c) Marty's mass can be determined by measuring the period of vibration of the oscillating chair (displacement is irrelevant) and using the equation ($T = 2\pi\sqrt{m/k}$).
(d) The period of oscillation would decrease (no change in displacement).
(e) Inertial mass affects an object's response to a non-gravitational force as described by Newton's 2nd Law of Motion. In this situation, the force is due to the spring and the response is the period of oscillation.

EK: 1.C.1 Inertial mass is the property of an object or a system that determines how its motion changes when it interacts with other objects or systems. 1.C.3 Objects and systems have properties of inertial mass and gravitational mass that are experimentally verified to be the same and that satisfy conservation principles.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 1.C.1.1 The student is able to design an experiment for collecting data to determine the relationship between the net force exerted on an object, its inertial mass, and its acceleration. 1.C.3.1 The student is able to design a plan for collecting data to measure gravitational mass and inertial mass, and to distinguish between the two experiments.

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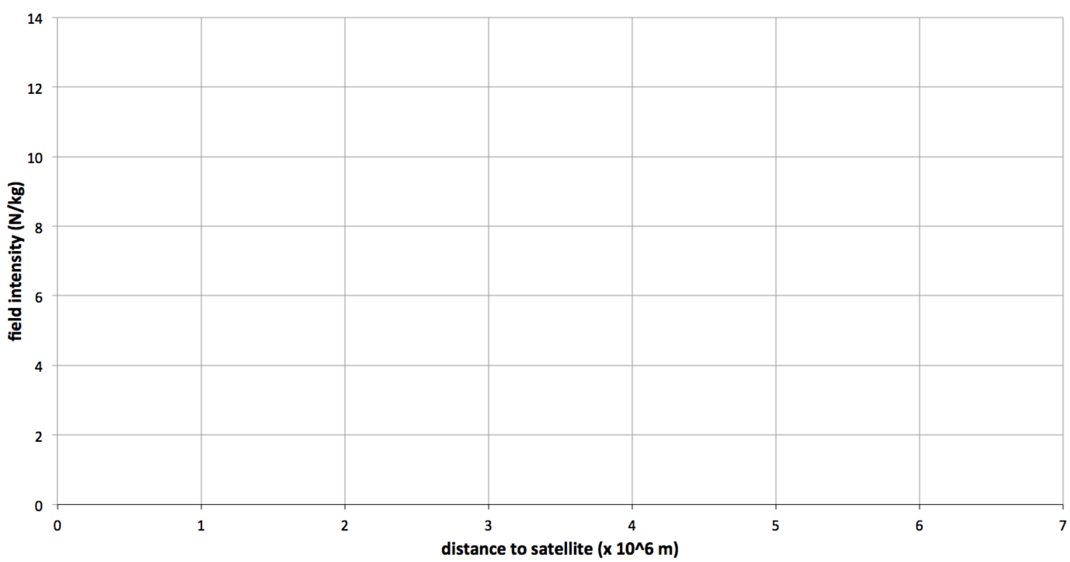
6. A space probe is sent on a mission to map out the gravitational field intensity in the vicinity of a satellite of planet X. Some of the data collected is shown in the chart below:

distance to satellite ($\times 10^6$ m)	field intensity (N/kg)
2.0	13.3
2.5	8.4
3.0	5.9
3.5	4.5
4.0	3.3
6.0	1.5

(a) On the axes below, plot the gravitational field intensity, g, vs. the distance, R, to the satellite.
(b) Draw in the appropriate best fit line or curve.

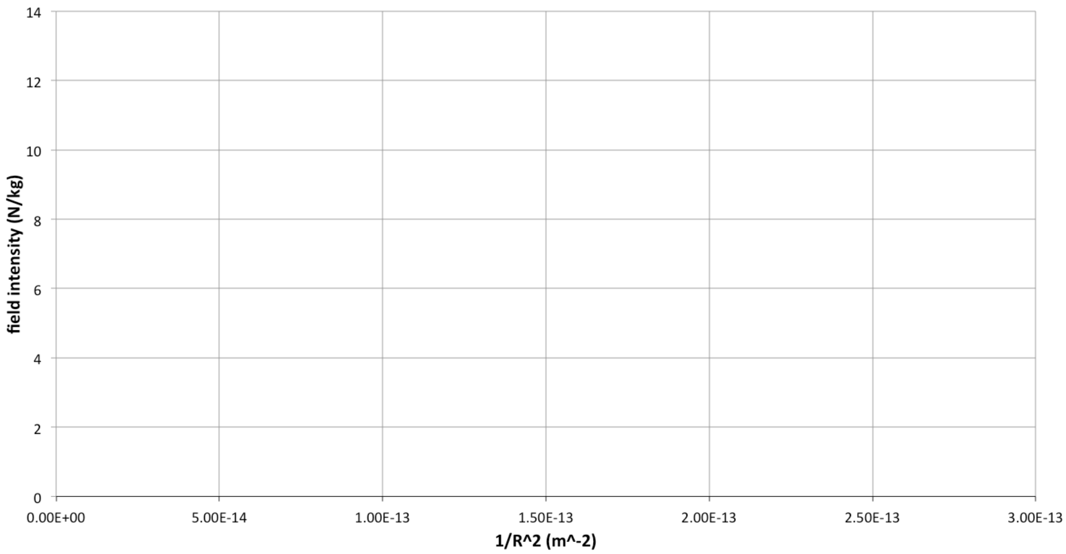


(c) Using the best fit, what distance corresponds to a field intensity of 2.1 N/kg?

(d) In order to determine the mass of the satellite, a plot of field intensity vs. $1/R^2$ can be utilized. Fill in the appropriate values for $1/R^2$ in the chart below.

$1/R^2$ ($1/m^2$)	field intensity (N/kg)
	13.3
	8.4
	5.9
	4.5
	3.3
	1.5

(e) On the axes below plot gravitational field intensity, g, vs. $1/R^2$.



(f) Use the plot and best fit to determine the mass of the planet.

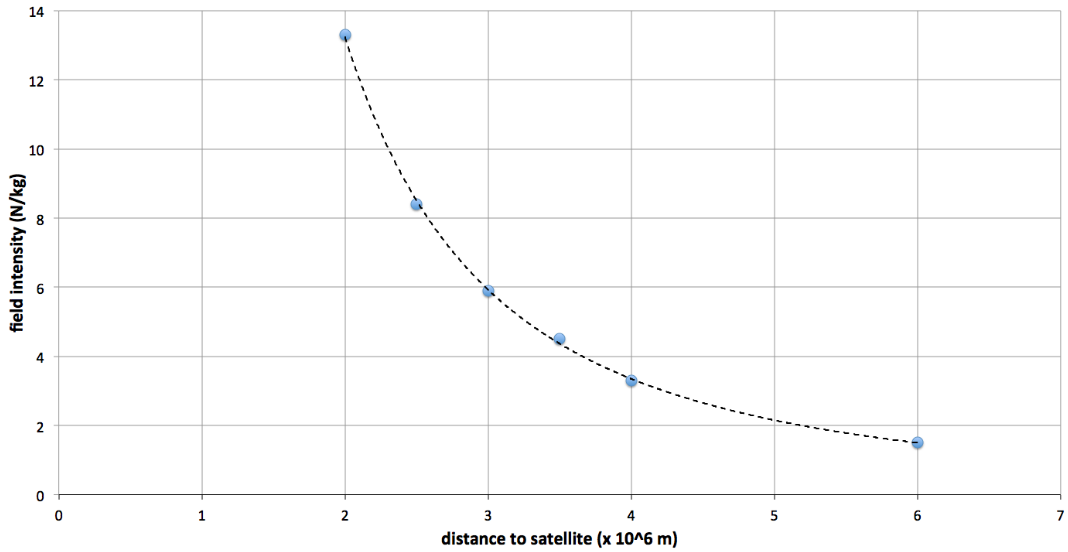
EK: 2.A.1 A vector field gives, as a function of position (and perhaps time), the value of a physical quantity that is described by a vector. 2.B.2 The gravitational field caused by a spherically symmetric object with mass is radial and, outside the object, varies as the inverse square of the radial distance from the center of that object.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 2.3 The student can estimate numerically quantities that describe natural phenomena. 5.1 The student can analyze data to identify patterns or relationships. 5.3 The student can evaluate the evidence provided by data sets in relation to a particular scientific question.

LO: 2.B.2.1 The student is able to apply g=GM/r^2 to calculate the gravitational field due to an object with mass M, where the field is a vector directed toward the center of the object of mass M. 2.B.2.2 The student is able to approximate a numerical value of the gravitational field (g) near the surface of an object from its radius and mass relative to those of the Earth or other reference objects.

Answer:

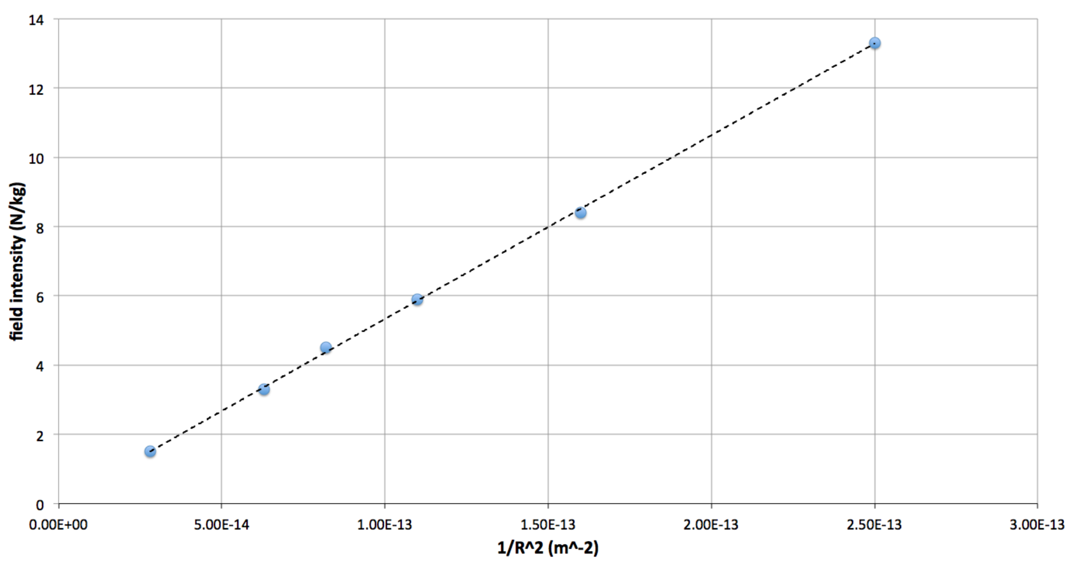
(a)
(b)



(c) distance of 5.0 ± .2 x 10^6 meters (corresponds to g = 2.1 N/kg)
(d)

$1/R^2$ ($1/m^2$)	field intensity (N/kg)
2.5×10^{-13}	13.3
1.6×10^{-13}	8.4
1.1×10^{-13}	5.9
8.2×10^{-14}	4.5
6.3×10^{-14}	3.3
2.8×10^{-14}	1.5

(e)



(f) slope = $gR^2 = 5\times10^{13}$ m^3/s^2. If $g = Gm_p/R^2$ then $m_p = (gR^2)/G$ = slope/G = 7.5×10^{23} kg